FHSST Authors

The Free High School Science Texts: Textbooks for High School Students Studying the Sciences Physics Grades 10 - 12

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Chapter 22

Mechanical Properties of Matter - Grade 12

22.1 Introduction

In this chapter we will look at some mechanical (physical) properties of various materials that we use. The mechanical properties of a material are those properties that are affected by forces being applied to the material. These properties are important to consider when we are constructing buildings, structures or modes of transport like an aeroplane.

22.2 Deformation of materials

22.2.1 Hooke's Law

Deformation (change of shape) of a solid is caused by a force that can either be compressive or tensile when applied in one direction (plane). Compressive forces try to compress the object (make it smaller or more compact) while tensile forces try to tear it apart. We can study these effects by looking at what happens when you compress or expand a spring.

Hooke's Law describes the relationship between the force applied to a spring and its extension.

Historical Note: Hooke's Law
Hooke's law is named after the seventeenth century physicist Robert Hooke who discovered it in 1660 (18 July 1635 - 3 March 1703).

Definition: Hooke's Law
In an elastic spring, the extension varies linearly with the force applied.
$F = -kx$ where F is the force in newtons (N), k is the spring constant in $N \cdot m^{-1}$ and x is the extension in metres (m).

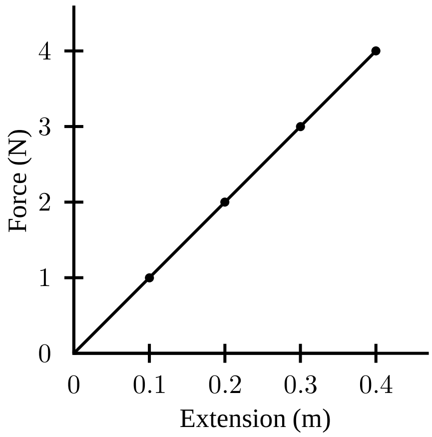

Figure 22.1: Hooke's Law - the relationship between extension of a spring and the force applied to it.

Activity :: Experiment : Hooke's Law

Aim:

Verify Hooke's Law.

Apparatus:

- weights
- spring
- ruler

Method:

1. Set up a spring vertically in such a way that you are able to hang weights from it.
2. Measure the extension of the spring for a range of different weights.
3. Draw a table of force (weight) in newtons and corresponding extension.
4. Draw a graph of force versus extension for your experiment.

Conclusions:

1. What do you observe about the relationship between the applied force and the extension?
2. Determine the gradient of the graph.
3. Hence, calculate the spring constant for your spring.

Worked Example 146: Hooke's Law I

Question: A spring is extended by 7 cm by a force of 56 N. Calculate the spring constant for this spring.

Answer

$$\begin{aligned} F &= -kx \\ 56 &= -k.0{,}07 \end{aligned}$$

$$\begin{aligned} k &= \frac{-56}{0{,}07} \\ &= -800 \text{ N.m}^{-1} \end{aligned}$$

Worked Example 147: Hooke's Law II

Question: A spring of length 20cm stretches to 24cm when a load of 0,6N is applied to it.

1. Calculate the spring constant for the spring.
2. Determine the extension of the spring if a load of 0,5N is applied to it.

Answer

1.

$$\begin{aligned} x &= 24 \text{ cm} - 20 \text{ cm} \\ &= 4 \text{ cm} \\ &= 0{,}04 \text{ m} \end{aligned}$$

$$\begin{aligned} F &= -kx \\ 0{,}6 &= -k.\,0{,}04 \end{aligned}$$

$$k = -15 \text{ N.m}^{-1}$$

2.

$$F = -kx$$

$$\begin{aligned} x &= \frac{F}{-k} \\ &= \frac{0{,}5}{15} \\ &= 0{,}033 \text{ m} \\ &= 3{,}3 \text{ cm} \end{aligned}$$

Worked Example 148: Hooke's Law III

Question: A spring has a spring constant of -400 N.m^{-1}. By how much will it stretch if a load of 50 N is applied to it?
Answer

$$\begin{aligned} F &= -kx \\ 50 &= -(-400)x \end{aligned}$$

$$\begin{aligned} x &= \frac{50}{400} \\ &= 0{,}125 \text{ m} \\ &= 12{,}5 \text{ cm} \end{aligned}$$

22.2.2 Deviation from Hooke's Law

We know that if you have a small spring and you pull it apart too much it stops 'working'. It bends out of shape and loses its springiness. When this happens Hooke's Law no longer applies, the spring's behaviour deviates from Hooke's Law.

Depending on what type of material we are dealing the manner in which it deviates from Hooke's Law is different. We give classify materials by this deviation. The following graphs show the relationship between force and extension for different materials and they all deviate from Hooke's Law. Remember that a straight line show proportionality so as soon as the graph is no longer a straight line, Hooke's Law no longer applies.

Brittle material

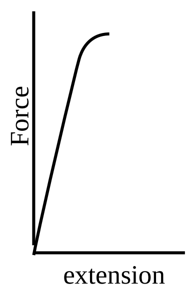

Figure 22.2: A hard, brittle substance

This graph shows the relationship between force and extension for a brittle, but strong material. Note that there is very little extension for a large force but then the material suddenly fractures. Brittleness is the property of a material that makes it break easily without bending.

Have you ever dropped something made of glass and seen it shatter? Glass does this because of its brittleness.

Plastic material

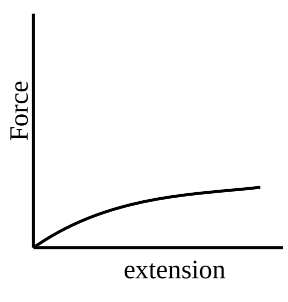

Figure 22.3: A plastic material's response to an applied force.

Here the graph shows the relationship between force and extension for a plastic material. The material extends under a small force but it does not fracture.

Ductile material

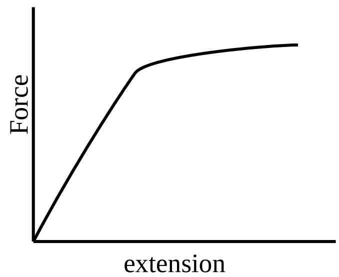

Figure 22.4: A ductile substance.

In this graph the relationship between force and extension is for a material that is ductile. The material shows plastic behaviour over a range of forces before the material finally fractures. Ductility is the ability of a material to be stretched into a new shape without breaking. Ductility is one of the characteristic properties of metals.

A good example of this is aluminium, many things are made of aluminium. Aluminium is used for making everything from cooldrink cans to aeroplane parts and even engine blocks for cars. Think about squashing and bending a cooldrink can.

Brittleness is the opposite of ductility.

When a material reaches a point where Hooke's Law is no longer valid, we say it has reached its *limit of proportionality*. After this point, the material will not return to its original shape after the force has been removed. We say it has reached its *elastic limit*.

Definition: Elastic limit
The elastic limit is the point beyond which permanent deformation takes place.

Definition: Limit of proportionality
The limit of proportionality is the point beyond which Hooke's Law is no longer obeyed.

Exercise: Hooke's Law and deformation of materials

1. What causes deformation?
2. Describe Hooke's Law in words and mathematically.
3. List similarities and differences between ductile, brittle and polymeric materials, with specific reference to their force-extension graphs.
4. Describe what is meant by the *elastic limit*.
5. Describe what is meant by the *limit of proportionality*.
6. A spring of length 15 cm stretches to 27 cm when a load of 0,4 N is applied to it.
   A Calculate the spring constant for the spring.
   B Determine the extension of the spring if a load of 0,35 N is applied to it.
7. A spring has a spring constant of -200 N.m^{-1}. By how much will it stretch if a load of 25 N is applied to it?
8. A spring of length 20 cm stretches to 24 cm when a load of 0,6 N is applied to it.

A Calculate the spring constant for the spring.
B Determine the extension of the spring if a load of 0,8 N is applied to it.

22.3 Elasticity, plasticity, fracture, creep

22.3.1 Elasticity and plasticity

Materials are classified as plastic or elastic depending on how they respond to an applied force. It is important to note that plastic substances are not necessarily a type of plastic (polymer) they only behave like plastic. Think of them as being like plastic which you will be familiar with.

A rubber band is a material that has elasticity. It returns to its original shape after an applied force is removed, providing that the material is not stretched beyond its elastic limit.

Plasticine is an example of a material that is plastic. If you flatten a ball of plasticine, it will stay flat. A plastic material does not return to its original shape after an applied force is removed.

- Elastic materials return to their original shape.
- Plastic materials deform easily and do not return to their original shape.

22.3.2 Fracture, creep and fatigue

Some materials are neither plastic nor elastic. These substances will break or fracture when a large enough force is applied to them. The brittle glass we mentioned earlier is an example.

Creep occurs when a material deforms over a long period of time because of an applied force. An example of creep is the bending of a shelf over time when a heavy object is put on it. Creep may eventually lead to the material fracturing. The application of heat may lead to an increase in creep in a material.

Fatigue is similar to creep. The difference between the two is that fatigue results from the force being applied and then removed repeatedly over a period of time. With metals this results in failure because of metal fatigue.

- Fracture is an abrupt breaking of the material.
- Creep is a slow deformation process due to a continuous force over a long time.
- Fatigue is weakening of the material due to short forces acting many many times.

?

Exercise: Elasticity, plasticity, fracture and creep

1. List the similarities and differences between elastic and plastic deformation.
2. List the similarities and differences between creep and fracture as modes of failure in material.

22.4 Failure and strength of materials

22.4.1 The properties of matter

The strength of a material is defined as the stress (the force per unit cross-sectional area) that it can withstand. Strength is measured in newtons per square metre $(N \cdot m^{-2})$.

Stiffness is a measure of how flexible a material is. In Science we measure the stiffness of a material by calculating its Young's Modulus. The Young's modulus is a ratio of how much it bends to the load applied to it. Stiffness is measure in newtons per metre $(N \cdot m^{-1})$.

Hardness of a material can be measured by determining what force will cause a permanent deformation in the material. Hardness can also be measured using a scale like Mohs hardness scale. On this scale, diamond is the hardest at 10 and talc is the softest at 1.

Remembering that the Mohs scale is the hardness scale and that the softest substance is talc will often come in handy for general knowledge quizes.

The *toughness* of a material is a measure of how it can resist breaking when it is stressed. It is scientifically defined as the amount of energy that a material can absorb before breaking.

A ductile material is a substance that can undergo large plastic deformation without fracturing. Many metals are very ductile and they can be drawn into wires, e.g. copper, silver, aluminium and gold.

A *malleable* material is a substance that can easily undergo plastic deformation by hammering or rolling. Again, metals are malleable substances, e.g. copper can be hammered into sheets and aluminium can be rolled into aluminium foil.

A brittle material fractures with very little or no plastic deformation. Glassware and ceramics are brittle.

22.4.2 Structure and failure of materials

Many substances fail because they have a weakness in their atomic structure. There are a number of problems that can cause these weaknesses in structure. These are vacancies, dislocations, grain boundaries and impurities.

Vacancies occur when there are spaces in the structure of a crystalline solid. These vacancies cause weakness and the substance often fail at these places. Think about bricks in a wall, if you started removing bricks the wall would get weaker.

Dislocations occur when there are no strong bonds between two rows in a crystal lattice. The crystal will fail along this boundary when sufficient force is applied. The two pieces of the crystal keep their shape and structure but move along the boundary.

Impurities in a crystal structure can cause a weak spot in the crystal lattice around the impurity. Like vacancies, the substance often fail from these places in the lattice. This you can think of as bricks in a wall which don't fit properly, they are the wrong kind of bricks (atoms) to make the structure strong.

A difference in *grain size* in a crystal lattice will result in rusting or oxidation at the boundary which again will result in failure when sufficient force is applied.

22.4.3 Controlling the properties of materials

There are a number of processes that can be used to ensure that materials are less likely to fail. We shall look at a few methods in this section.

Cold working

Cold working is a process in which a metal is *strengthened* by repeatedly being reshaped. This is carried out at a temperature below the melting point of the metal. The repeated shaping of the metal result in dislocations which then prevent further dislocations in the metal. Cold working increases the strength of the metal but in so doing, the metal loses its ductility. We say the metal is *work-hardened*.

Annealing

Annealing is a process in which a metal is heated strongly to a temperature that is about half of its melting point. When the metal cools, it recrystallises which removes vacancies and dislocations in the metal. Annealing is often used before cold working. In annealing the metal cools very very slowly.

Alloying

An alloy is a mixture of a metal with other substances. The other substances can be metal or non-metal. An alloy often has properties that are very different to the properties of the substances from which it is made. The added substances strengthen the metal by preventing dislocations from spreading. Ordinary steel is an alloy of iron and carbon. There are many types of steel that also include other metals with iron and carbon. Brass is an alloy of copper and Zinc. Bronze is an alloy of copper and tin. Gold and silver that is used in coins or jewellery are also alloyed.

Tempering

Tempering is a process in which a metal is melted then quickly cooled. The rapid cooling is called quenching. Usually tempering is done a number of times before a metal has the correct properties that are needed for a particular application.

Sintering

Sintering is used for making ceramic objects among other things. In this process the substance is heated so that its particles stick together. It is used with substances that have a very high melting point. The resulting product is often very pure and it is formed in the process into the shape that is wanted. Unfortunately, sintered products are brittle.

22.4.4 Steps of Roman Swordsmithing

- Purifying the iron ore.
- Heating the iron blocks in a furnace with charcoal.
- Hammering and getting into the needed shape. The smith used a hammer to pound the metal into blade shape. He usually used tongs to hold the iron block in place.
- Reheating. When the blade cooled, the smith reheated it to keep it workable. While reheated and hammered repeatedly.
- *Quenching* which involved the process of white heating and cooling in water. Quenching made the blade harder and stronger. At the same time it made the blade quite brittle, which was a considerable problem for the sword smiths.
- *Tempering* was then done to avoid brittleness the blade was tempered. In another words it was reheated a final time to a very specific temperature. How the Romans do balanced the temperature? The smith was guided only by the blade's color and his own experience.

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Exercise: Failure and strength of materials

1. List the similarities and differences between the brittle and ductile modes of failure.
2. What is meant by the following terms:
   A vacancies
   B dislocations
   C impurities
   D grain boundaries
3. What four terms can be used to describe a material's mechanical properties?
4. What is meant by the following:
   A cold working
   B annealing
   C tempering
   D introduction of impurities
   E alloying
   F sintering

22.5 Summary

1. Hooke's Law gives the relationship between the extension of a spring and the force applied to it. The law says they are proportional.
2. Materials can be classified as plastic or elastic depending on how they respond to an applied force.
3. Materials can fracture or undergo creep or fatigue when forces are applied to them.
4. Materials have the following mechanical properties to a greater or lesser degree: strength, hardness, ductility, malleability, brittleness, stiffness.
5. Materials can be weakened by have the following problems in their crystal lattice: vacancies, dislocations, impurities, difference in grain size.
6. Materials can have their mechanical properties improved by one or more of the following processes: cold working, annealing, adding impurities, tempering, sintering.

22.6 End of chapter exercise

1. State Hooke's Law in words.
2. What do we mean by the following terms with respect to Hooke's Law?
   A elastic limit
   B limit of proportionality
3. A spring is extended by 18 cm by a force of 90 N. Calculate the spring constant for this spring.
4. A spring of length 8 cm stretches to 14 cm when a load of 0,8 N is applied to it.
   A Calculate the spring constant for the spring.
   B Determine the extension of the spring if a load of 0,7 N is applied to it.

5. A spring has a spring constant of -150 N.m^{-1}. By how much will it stretch if a load of 80 N is applied to it?
6. What do we mean by the following terms when speaking about properties of materials?
   A hardness
   B toughness
   C ductility
   D malleability
   E stiffness
   F strength
7. What is Young's modulus?
8. In what different ways can we improve the material properties of substances?
9. What is a metal alloy?
10. What do we call an alloy of:
    A iron and carbon
    B copper and zinc
    C copper and tin
11. Do some research on what added substances can do to the properties of steel. Present you findings in a suitable table.

Appendix A

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